Wednesday: lecture

Thursday: Tutorial

Office hours →

Today

4-5 online

or in office

CSE525 Lec24 Approximation

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Debajyoti Bera (M21)

Set Cover $(U = \{x_1, \ldots x_n\}, T = \{S_1, S_2, \ldots, S_m\})$

→ fewest subsets that cover (contain) all elements in U.

C, B, A, D

$x^1 = 4, \quad x^2 = 2, \quad x^3 = 1, \quad x^4 = 1$

Greedy Set Cover (U,T={S1...Sm})

1. SC = { }
2. Until U has some uncovered element :
   a. Choose Si with the largest number of uncovered elements
   b. Add Si to SC
   c. Mark elements of Si as covered
3. return SC

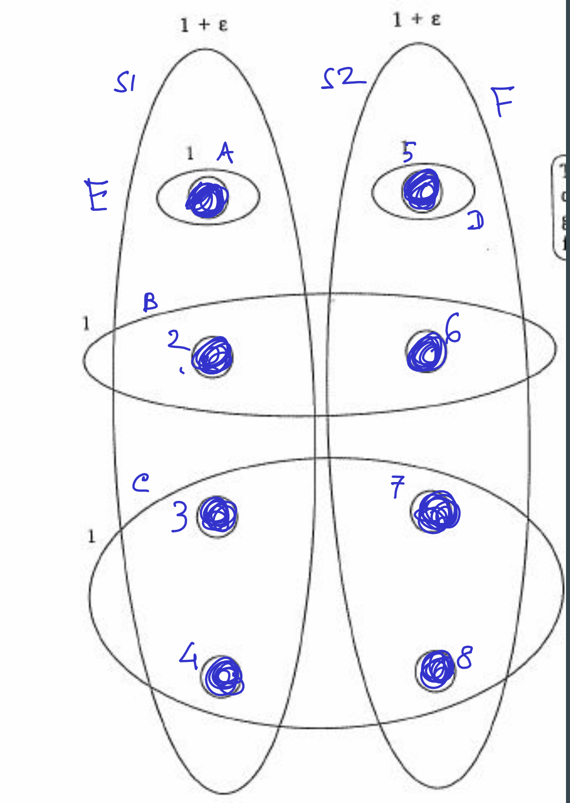
① Algo returns a set cover, and is polytime (n, nm)

② Compute approximation ratio

$OPT \leq APPROX \leq x \cdot OPT$

$APPROX \leq ??$

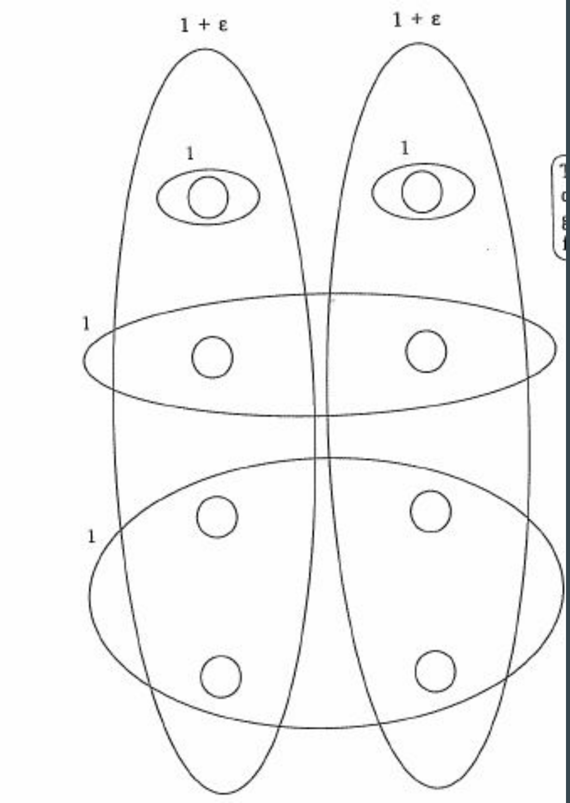
$?? \leq OPT$



Greedy Set Cover (U,T={S1...Sm})

U^i : universe after the ith iteration

1. SC = { }, $U^0 = U$, i=1
2. Until U has some element :
   a. S^i = Choose Si with the largest number of elements
   b. Add S^i to SC
   c. Remove elements of S^i from U and the other sets
   d. $r^i = |S^i|$, i ++
3. return SC



$|U^1| \geq |U^{OPT}|$

$|U^2| \geq |U^{OPT}|$

Greedy Set Cover (U,T={S1…Sm})

APPROX $\geq$ OPT

1. AP SC = { }, U^0 = U, i=0
2. Until U has some element :
   a. i++
   b. L^i Choose Si with the largest number of elements

      original subset corresponding to L^i

   c. Add L^i to SC
   d. Remove elements of L^i from U^{i-1} and the other sets → U^i, $S1^i$, $S2^i$, … Sm^i

      set system after ith iteration

   e. $r^i = |L^i|$
3. return APSC (let APPROX = i)

   ↳ # iterations

Optimum : OPTSC (with OPT sets)

{ S_3, S_6, S_9 }

S_3^i S_6^i S_9^i cover all elements of U^i

* $r^1 + r^2 + \ldots + r^{APPROX} = ?$ |U|

Thm: $r^1 + r^2 + \ldots + r^{OPT} = ?$ # elements removed in the first opt rounds $\leq |U|$

For any iteration i = 1 … OPT, …

1. OPTSC is also a valid cover for ith iteration
2. $\sum |S^i| >=$ _____ where $\sum$ is over all subsets in OPTSC $|A^1| + |C^1| + |E^1| \geq |U^1|$
3. $|L^{i+1}| >= |S^i|$ for any subset S^i in OPTSC
4. $|L^{i+1}| >= |U^i| / OPT \geq \frac{|U^{opt}|}{OPT}$ $|L^2| \geq |A^1|$
5. $r^{i+1} = |L^{i+1}| >= |U^{OPT}| / OPT$ $|L^2| \geq |C^1|$
6. $r^1 + r^2 + \ldots + r^{OPT} >= |U^{OPT}|$ $|L^2| \geq |E^1|$ $\geq |U^1|$
7. $|U^{OPT}| = |U| - r^1 - r^2 - \ldots - r^{OPT}$ $|L^2| \geq \frac{|A^1| + |C^1| + |E^1|}{OPT}$

$S1^i$ $S2^i$ … Sm^i U^i

$L^5 \geq |S^4|$ for any S in OPTSC

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $A = \{1, 2\}$ $B = \{2, 4, 6, 8\}$ $C = \{7, 9, 3\}$ $D = \{1, 6, 8\}$

$E = \{4, 5, 6, 7, 8\}$ $F = \{2, 3, 9\}$

OPT SC = A, C, E

$U^0 = U$

$L^1 = E$, $U^1 = \{1, 2, 3, 9\}$, $A^1 = \{1, 2\}$ $B^1 = \{2\}$ $C^1 = \{9, 3\}, \ldots$

$E^1 = \phi$

$A^1\ C^1\ E^1$ also cover U^1 (Exercise)

$*$ $\quad 7 \to\ r^1 + r^2 + \cdots + r^{OPT} = |U| - |U^{OPT}|$

$+6 \to\ 2(r^1 + \cdots + r^{OPT}) \geq |U|$

$\Rightarrow\ r^1 + \cdots + r^{OPT} \geq \frac{|U|}{2}$ } # elements removed in first OPT rounds $\geq 1/2 * |U|$

$r^{i+1} \geq |U^{OPT}| / OPT$ $\qquad \sum_{i=0}^{OPT-1} r^{i+1} \geq |U^{OPT}|$